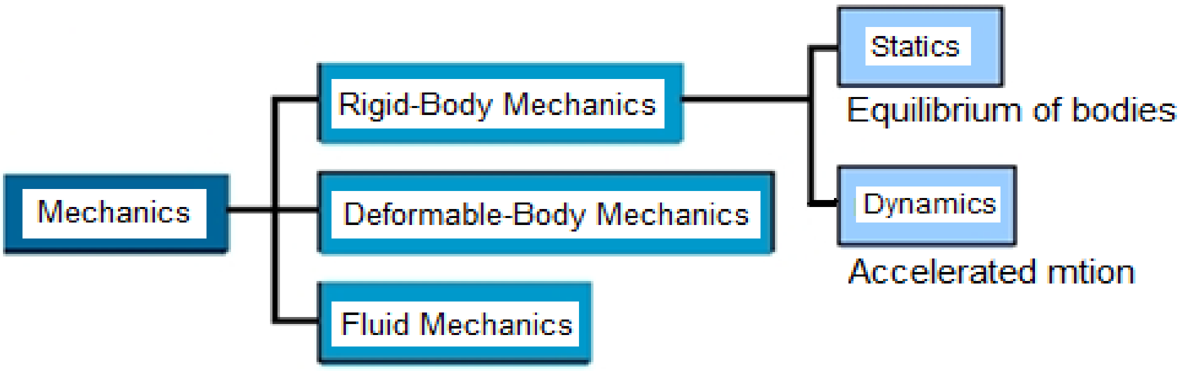
# Introduction

**Mechanics** is the branch of physics concerned with the behavior of physical bodies at rest or in motion when subjected to forces or displacements, and the subsequent effects of the bodies on their environment. It can be divided to the following:




**Statics** is the study of bodies that are at rest or move with constant velocity. We can consider statics as a special case of dynamics, in which the acceleration is zero

## Fundamental Concepts:

**Length:** Length is the quantity used to describe the position of a point in space relative to another point.

**Time:** Time is the interval between two events.

**Mass:** The amount of matter contained in a body. The mass of a body determines both the action of gravity on the body, and the resistance to changes in motion. This resistance to changes in motion is referred to as inertia.

**Force:** Force is the action of one body on another. Force may or may not be the result of direct contact between bodies such as the gravitational, and electromagnetic

**Rigid Body:** Rigid body - a body is considered rigid when the relative movements between its parts are negligible.

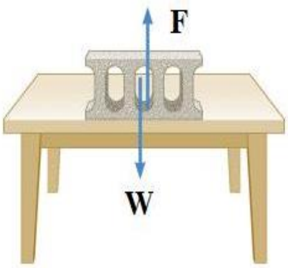
## Newton's 3 Fundamental Laws

1st Law - A particle remains at rest or continues to move in a straight line with a constant speed if there is no unbalanced force acting on it (resultant force = 0).

2nd Law - The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.

$F = m \cdot a$ where: F: force
m: Mass
a: Acceleration

3rd Law - The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and act along the same line of action (Collinear).



## Units:

| | | | | | |
|---|---|---|---|---|---|
| | | SI | | U.S. | |
| Mass | M | kilogram | kg | slug | |
| Length | L | meter | m | feet | ft |
| Force | F | Newton | N | pound | lb |
| Time | T | second | s | second | sec |

## Mass vs. Weight:

The weight of a body is the force exerted on the body due to gravitational attraction of the Earth.

### W= m g

W is weight

m is mass

g is acceleration due to gravity $= 9.81\ m/s^2$ (SI units)

Also $g = 32.2\ ft/s^2$ (English units)

$1\ N = (1\ kg)\ (1\ m/s^2)$

$1\ lb = (1\ slug)\ (1\ ft/sec^2)$

Weight is a force.

Example: The weight of 1 kg Mass is:

$W = mg$

$W = (1\ kg)\ (9.81\ m/\ s^2)$

$W = 9.81\ N$

## Unit conversion:

| | | |
|---|---|---|
| l lb | = | 4.4482 N |
| 1slug | = | 1 lb $s^2$ /ft = 14.5938 kg |
| 1 ft | = | 0.3048 m |
| 1 ft | = | 12 in |
| 1mile | = | 5,280 ft |
| 1 kip | = | 1,000 lb |
| 1 ton | = | 2,000 lb |

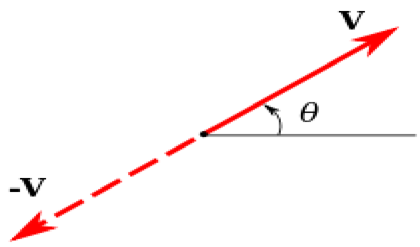
## Scalars and Vectors

There are two types of quantities in physics Scalars and Vectors.

**Scalars** quantities has only *magnitude* such as length, area, volume, mass, density, temperature, energy, ..etc

**Vectors** has *magnitude + direction* such as: displacement, direction velocity, force,..etc

## Vectors:

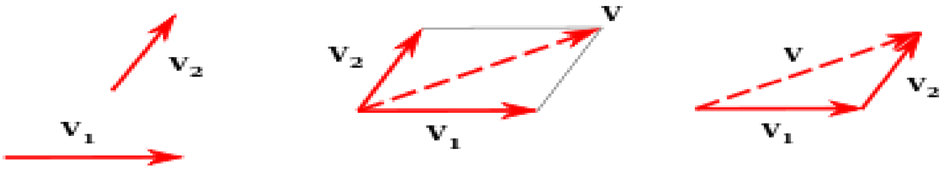



A vector quantity $\mathbf{V}$ is drawn as in Figure 1.1, a line segment with an arrow head to indicate direction. The length of the line segment indicates the vector magnitude, $|V|$ (printed as $V$), using a convenient scale.

Figure 1.1: Vector drawing and labeling conventions

A vector's direction may be described by an angle, given from a known origin and line of reference, as in Figure 1.1.

### Using Vectors




(*a*) (*b*) (*c*)

Figure 1.2: Parallelogram law of combination

Vectors obey the parallelogram law of combination

$$\mathbf{V} = \mathbf{V_1} + \mathbf{V_2}, \qquad \mathbf{V_1} + \mathbf{V_2} = \mathbf{V_2} + \mathbf{V_1}.$$

The *vector sum* is indicated by the addition sign between bold-faced vectors. This should not be confused with the *scalar sum* of the two vectors, $V_1 + V_2$. Because of the geometry of the parallelogram, clearly $V \neq V_1 + V_2$.

Typically we use *rectangular components* using $x$ and $y$ as coordinates.

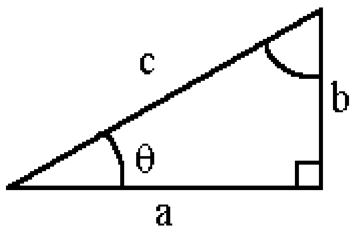

*(a)* *(b)*

Figure 1.4: Parallelogram law of combination

$$\theta = \tan^{-1}\frac{V_y}{V_x}$$

Triangle law can be used in calculating vectors angles which is as folowing:

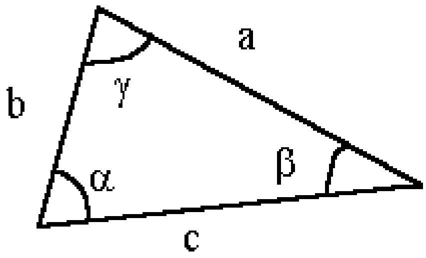
**For a right triangle:**




$a^2 + b^2 = c^2$

$\tan(\theta) = b/a$, $\sin(\theta) = b/c$, $\cos(\theta) = a/c$

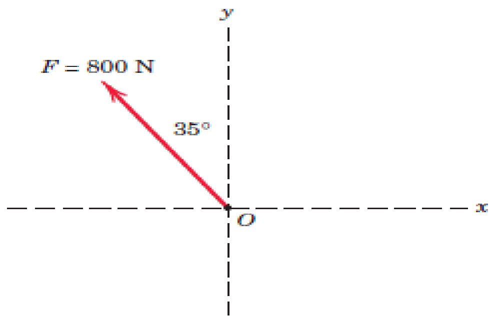
## For a general triangle:




$\alpha+\beta+\gamma=180^{o}$

Sine law: $\dfrac{a}{\sin(\alpha)}=\dfrac{b}{\sin(\beta)}=\dfrac{c}{\sin(\gamma)}$

Cosine law: $c=\sqrt{a^2+b^2-2ab\cos(\gamma)}$




<u>Example:</u> The force **F** has a magnitude of 800 N. Express **F** as a vector in terms of the unit vectors **i** and **j**. Identify the *x* and *y* scalar components of **F**.

Solution:

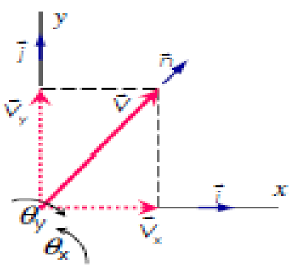
$$\begin{cases} F_x = -800 \sin 35^\circ = -459 \text{ N} \\ F_y = 800 \cos 35^\circ = 655 \text{ N} \end{cases}$$

$$\bar{F} = -459\,i + 655\,j \quad \text{N}$$

**<u>Unit Vector:</u>**

$$\bar{\mathbf{V}} = |\,\mathbf{V}\,|\;\bar{n}$$

Magnitude Direction (dimensionless)




$\bar{n}$ = Unit vector in direction of $\bar{V}$

$$\bar{n}=\frac{\bar{V}}{|\bar{V}|}=\frac{V_x\hat{i}+V_y\hat{j}}{|\bar{V}|}=\frac{V_x}{|\bar{V}|}\hat{i}+\frac{V_y}{|\bar{V}|}\hat{j}$$

$$=\cos\theta_x\hat{i}+\cos\theta_y\hat{j}$$

$$\boxed{\frac{V_x}{|\bar{V}|}=\cos\theta_x=\text{direction cosine}}$$

$$\cos\theta_x^{\,2}+\cos\theta_y^{\,2}=1$$

Taking $V$ as the vector magnitude, a vector may be described as its magnitude multiplied by its unit vector. The value of $\mathbf{V}$ does not change as a result, because the *unit vector* indicates the proper direction and has a magnitude of one.

In order to describe a vector in 2 or 3 (orthogonal) dimensions, we use the unit vectors, $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ to indicate a vector's direction towards $x$, $y$, and $z$, respectively (as seen in Figure 1.5).

$\bar{\mathbf{V}}$ = $V$x **i** + $V$y **j** + $V$z **k**

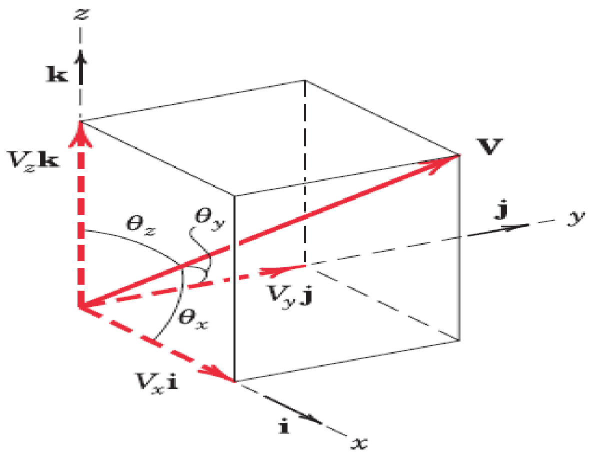



Figure 1.5

## Example:

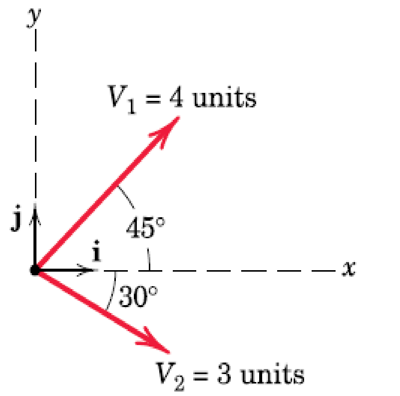
For the vectors $\bar{V}_1$ and $\bar{V}_2$ shown in the figure,

(*a*) determine the magnitude $S$ of their vector sum $\bar{S} = \bar{V}_1 + \bar{V}_2$

(*b*) determine the angle $\alpha$ between $\bar{S}$ and the positive $x$-axis

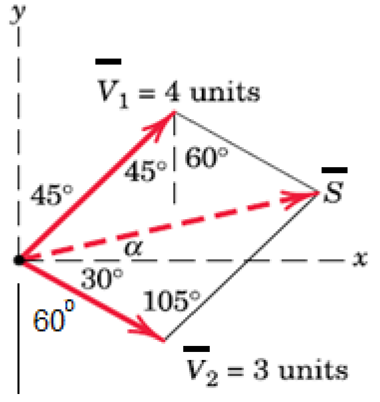
(*c*) write $\bar{S}$ as a vector in terms of the unit vectors **i** and **j** and then write a unit vector $\bar{n}$ along the vector sum $\bar{S}$

(*d*) determine the vector difference $\bar{D} = \bar{V}_1 - \bar{V}_2$




Solution:

(*a*) We construct to scale the parallelogram shown in Fig. *a* for adding $\mathbf{V}_1$ and $\mathbf{V}_2$. Using the law of cosines, we have



(*a*)

$$S^2 = 3^2 + 4^2 - 2(3)(4)\cos 105°$$

$$S = 5.59 \text{ units} \qquad \textit{Ans.}$$

(*b*) Using the law of sines for the lower triangle, we have

$$\frac{\sin 105°}{5.59} = \frac{\sin(\alpha + 30°)}{4}$$

$$\sin(\alpha + 30°) = 0.692$$

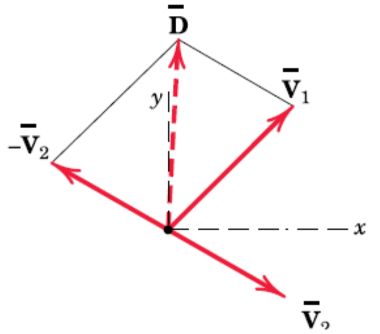
$$(\alpha + 30°) = 43.8° \qquad \alpha = 13.76° \qquad \textit{Ans.}$$

(*c*) With knowledge of both $S$ and $\alpha$, we can write the vector $\mathbf{S}$ as

$$\bar{\mathbf{S}} = S[\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha]$$

$$= S[\mathbf{i}\cos 13.76° + \mathbf{j}\sin 13.76°] = 5.43\mathbf{i} + 1.328\mathbf{j} \text{ units} \qquad \textit{Ans.}$$

Then $$\bar{\mathbf{n}} = \frac{\bar{\mathbf{S}}}{S} = \frac{5.43\mathbf{i} + 1.328\mathbf{j}}{5.59} = 0.971\mathbf{i} + 0.238\mathbf{j} \qquad \textit{Ans.}$$




(*d*) The vector difference $\bar{\mathbf{D}}$ is

$$\bar{\mathbf{D}} = \bar{\mathbf{V}}_1 - \bar{\mathbf{V}}_2 = 4(\mathbf{i}\cos 45° + \mathbf{j}\sin 45°) - 3(\mathbf{i}\cos 30° - \mathbf{j}\sin 30°)$$

$$= 0.230\mathbf{i} + 4.33\mathbf{j} \text{ units} \qquad \textit{Ans.}$$

The vector $\bar{\mathbf{D}}$ is shown in Fig. *b* as $\bar{\mathbf{D}} = \bar{\mathbf{V}}_1 + (-\bar{\mathbf{V}}_2)$.

# Examples & Solutions

Example:

What is the mass of an object that weighs 1 pound?

Solution: W = mg

$1\ \text{lb} = m\ (32.2\ \text{ft/ s}^2)$

$m = 1/32.2\ \text{lb s}^2/\text{ft} = 1/32.2\ \text{slug}$

Example:

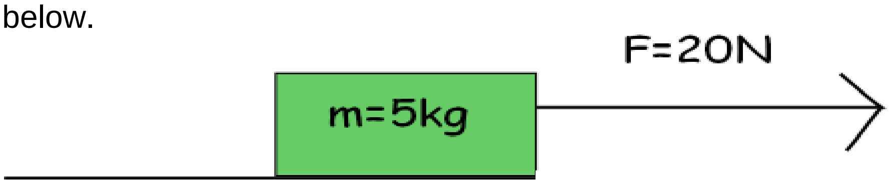
What is the weight of an object that has a mass of 1 slug?

*Solution:* $W = (1\ \text{lb s}^2 / \text{ft})(32.2\ \text{ft/ s}^2)$

$W = 32.2\ \text{lb}$

Example:

Where, *F* is the force and its unit is Newton, *m* is mass and has the unit kg and *a* is the acceleration has unit m/s². Find the acceleration of the block given in the picture below.




F=m.a
20N=5kg.a
a=4m/s.s

Example:

A force is specified by the vector $\bar{\mathbf{F}} = 160\mathbf{i} + 80\mathbf{j} - 120\mathbf{k}$ N. Calculate the angles made by $\bar{\mathbf{F}}$ with the positive *x*-, *y*-, and *z*-axes.

Solution:

$$F = \sqrt{160^2 + 80^2 + 120^2} = 215\ \text{lb}$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{160}{215} = 0.743, \quad \underline{\theta_x = 42.0^\circ}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{80}{215} = 0.371, \quad \underline{\theta_y = 68.2^\circ}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-120}{215} = -0.557, \quad \underline{\theta_z = 123.9^\circ}$$

Example:

Determine the angles made by the vector $\bar{\mathbf{V}}$ = -- 36**i** + 15**j** with the positive *x*- and *y*-axes.

Write the unit vector $\bar{\mathbf{n}}$ in the direction of $\bar{\mathbf{V}}$.

Solution:

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{36^2 + 15^2} = 39$$

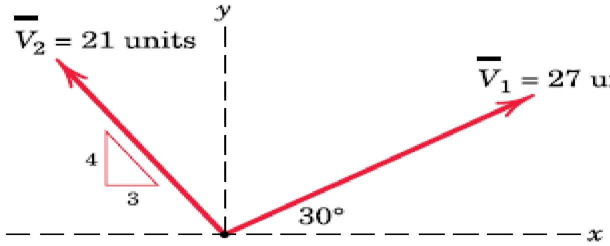
$$\cos\theta_x = \frac{V_x}{V} = \frac{-36}{39}, \quad \underline{\theta_x = 157.4^\circ}$$

$$\cos\theta_y = \frac{V_y}{V} = \frac{15}{39}, \quad \underline{\theta_y = 67.4^\circ}$$

$$\bar{n} = \frac{\bar{V}}{V} = \frac{-36\underline{i} + 15\underline{j}}{39} = -0.923\underline{i} + 0.385\underline{j}$$

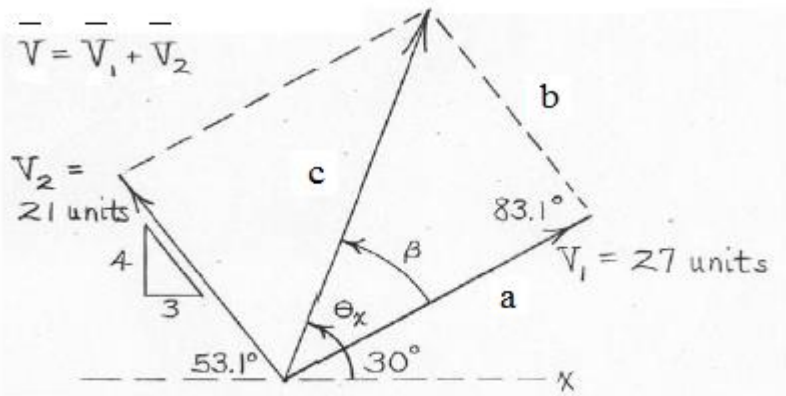
Example:

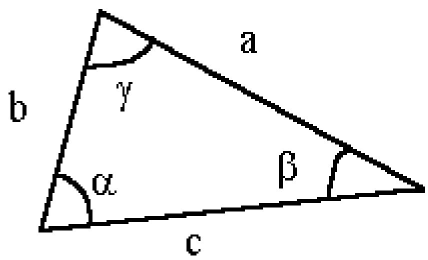
Determine the magnitude of the vector sum $\bar{\mathbf{V}}$ = $\bar{\mathbf{V}}$1 + $\bar{\mathbf{V}}$2 and the angle $\theta_x$ which **V** makes with the positive *x*-axis.




Solution:

From the general triangle law




$$c = \sqrt{a^2 + b^2 - 2ab\cos(\gamma)}$$

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

$$V^2 = 27^2 + 21^2 - 2(27)(21)\cos 83.1^\circ$$

$$V = \underline{32.2 \text{ units}}$$

$$\frac{\sin\beta}{21} = \frac{\sin 83.1^\circ}{32.2}, \quad \beta = 40.4^\circ$$

$$\theta_x = \beta + 30^\circ = 40.4^\circ + 30^\circ = \underline{70.4^\circ}$$